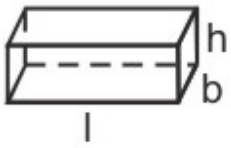
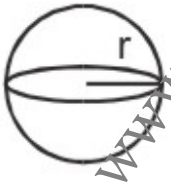
Volumes and Surface Areas

3 Dimensional Figures:

Three dimensional figures have volumes as they have all the three dimensions height, breadth, length. If you observe carefully, the base of a 3-dimensional figure is a 2-dimensional figure. If you multiply the area of the base with height, you get the volume. Area for three dimensional figures is called surface area. It is simply summation of the areas of several 2 dimensional figures on the 3 dimensional figures.

Name	Figure	Curved Surface area	Total surface area	Volume
Cuboid		2h(l+ b)	2(lb+ bh + lh)	lbh
Cube		$4a^2$	$6a^2$	a^3
Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	πr^2h
Right circular cone		πrl	$\pi r(l+ r)$	$\frac{1}{3}\pi r^2h$
Sphere		–	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$
Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$

1. Find the curved surface area of a hemisphere whose diameter is 14 cm.

Radius of hemisphere = $\frac{1}{2}$ x 14 cm = 7 cm.

Therefore, Curved surface Area = $2\pi r^2$ = 2 x $\frac{22}{7}$ x 7 x 7 = 308 cm^2

2. Find the height of a cuboid whose volume and base area are 144 m^3 and 18 m^2 respectively.

Height of the cuboid = $\frac{\text{Volume}}{\text{BaseArea}}$ = $\frac{144}{18}$ = 8 metre

3. Find the length of the longest pole that can be placed in a room 12 metre long, 8 metre broad and 9 metre high.

Diagonal of room = $\sqrt{\text{Length}^2+\text{Breadth}^2+\text{Height}^2}$ = $\sqrt{12^2+8^2+9^2}$ = $\sqrt{289}$ = 17 metre.

Therefore, Length of longest pole = 17 metre

4. The length, breadth and height of a room are 5 metre, 4 metre and 4 metre respectively. If all the four walls of the room are to be covered with 50 cm wide wall paper, find the length of the paper required.

Area of four walls = 2 x Height (Length + Breadth) = 2 x 4 x (5 + 4) = 72 m^2

Width of the wall paper = 50 cm = 0.5 metre

Therefore, Length of the paper required = 72 ÷ 0.5 = 72 ÷ $\frac{1}{2}$ = 72 x 2 = 144 metre

5. How many spheres of radius 2 cm can be made by melting two spheres of radius 12 cm each?

Radius of big sphere = 12 cm

Radius of smaller sphere = 2 cm

Ratio between radii of two spheres = 12 : 2 = 6 : 1

Ratio between volumes of two spheres = $6^3 : 1^3$ = 216 : 1

Therefore, 216 speres can be made from one big sphere.

Therefore, From two big spheres, smaller spheres made = 216 x 2 = 432

6. A cube of 9 cm a side is melted and 27 small cubes are made from that. Find side of the smaller cubes.

We know, Volume of cube = Side^3

Therefore, Volume of the small cube = $\frac{9^3}{27}$ = 27

Therefore, Side of the small cube = $\sqrt[3]{27}$ = 3 cm

7. A box is made of 1 cm thick wood. If outer dimensions of wood are 12 cm by 10 cm by 7 cm, find volume of wood.

Outer dimensions of box = 12 cm, 10 cm, 7 cm

Inner dimensions of box = 10 cm, 8cm, 5 cm

Therefore, Volume of wood = (12 x 10 x 7) cm^3 - (10 x 8 x 5)cm^3

= 840 cm^3 - 400 cm^3 = 440 cm^3.

8. How much metal is required to make a 20 metre long pipe, if its inner and outer diameter are 6 metre and 8 metre respectively.

Outer and inner radius of the pipe are 4 metre and 3 metre respectively.

Therefore, Quantity of metal required

= Outer volume of the pipe - Inner volume of the pipe

= $\pi(R^2-r^2)h$

= $\frac{22}{7}\times(4^2-3^2)$ x 20

= $\frac{22}{7}$ x 7 x 1 x 20 = 440 m^3

Hint: Pipe is a form of cylinder.

9. A wire is drawn from a solid iron sphere of radius 6cm. Find the length of the wire, if its diameter is 6 cm.

An iron wire is a form of cylinder.

Therefore, Volume of cylinder = Volume of Sphere

Therefore, $\pi\ (\text{Radius of cylinder})^{2}$ (Height of cylinder) = $\frac{4}{3}\pi\ (\text{Radius of sphere})^{3}$

Therefore, Height of the cylinder = $\frac{4}{3}$ x $\frac{\pi(\text{Radius of sphere})^{3}}{\pi(\text{Radius of cylinder})^{2}}$

= $\frac{4}{3} \times \frac{6 \times 6 \times 6}{3 \times 3}$ = 32 cm

Therefore, Length of the wire is 32 cm.

10. A field is 90 metre long and 50 metre broad. A 25 metre long, 20 metre broad and 4 metre deep tank dug in the field and the earth taken out is spread evenly over the remaining field. How much the level of field will rise?

Area of field = 90 x 50 = 4500 m^2

Area of field dug out = 25 x 20 = 500 m^2

Therefore, Area of remaining field = 4500 m^2 - 500 m^2 = 4000 m^2

Volume of the earth dug out = 25 x 20 x 4 = 2000 m^3

Therefore, Field will rise by $\frac{2000}{4000}$ = 0.5 metre

MCQ's

1. The volume of a sphere is $\frac{88}{21} \times 14^3 cm^3$ The curved surface of its sphere is :

a. 2424 cm^2

b. 2446 cm^2

c. 2464 cm^2

d. 2484 cm^2

Correct Option: C

Volume of the sphere = $\frac{4}{3}\pi r^3$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88}{21} \times (14)^3 \Rightarrow r = 14$

Curved Surface area = $4\pi r^2$ = $4 \times \frac{22}{7} \times 14^2$ = 2464 cm^2

2. If the radius of a sphere is doubled, then its volume is increased by :

a. 100%

b. 200%

c. 700%

d. 800%

Correct Option: D

Explanation:

Original volume = $\frac{4}{3}\pi r^3$

New volume = $\frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

Increase % = $(\frac{32}{3}\pi r^3 \times \frac{3}{4\pi r^3})\% = 800\%$

3. If the height of a cone is doubled, then its volume is increased by :

a. 100%

b. 200%

c. 300%

d. 400%

Correct Option: A

Explanation:

Original volume = $\frac{1}{3}\pi r^2 h$

New volume $\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$

Increase % $\left(\frac{\frac{2}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100\right)\% = 200\%$

4. The cost of painting the four walls of a room is Rs.350. The cost of painting a room three times in length, breadth and height will be :

a. Rs.1050

b. Rs.1400

c. Rs.3150

d. Rs.4200

Correct Option: C

Explanation:

Area of 4 walls of the room = $[2(l+b)\times h]m^2$

Area of 4 walls of new room = $[2(3l+3b)\times 3h]m^2$ = $9[2(l+b)\times h]m^2$

Cost of painting the 4 walls of new room = Rs.$(9\times 350) = Rs.3150$

5. The area of the base of a right circular cone is 154 cm^2 and its height is 14 cm. The curved surface of the cone is :

a. $(154\times\sqrt{5})cm^2$

b. 11 cm^2

c. $(154\times\sqrt{7})cm^2$

d. 5324 cm^2

Correct Option: A

Explanation:

$\frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = (154 \times \frac{7}{22})$ = 49 or r = 7 cm

Now, r = 7 and h = 14.

So, l = $\sqrt{(7)^2 + (14)^2}$

= $\sqrt{245} = 7\sqrt{5}$ cm

Area of curved surface = $\pi rl = (\frac{22}{7} \times 7 \times 7\sqrt{5})cm^2 = 154\sqrt{5}cm^2$

6. The material of a cone is converted into the shape of a cylinder of equal radius. If the height of the cylinder is 5 cm, the height of the cone is :

a. 10 cm

b. 15 cm

c. 18 cm

d. 24 cm

Correct Option: B

Explanation:

$\frac{1}{3}\pi r^2 \times h = \pi h^2 \times 5$ or h = 15 cm

7. A sold consists of a circular cylinder with an exact fitting right circular cone placed on the top. The height of the cone is h. If the total volume of the solid is three times the volume of the cone, then the height of the cylinder is :

a. 2 h

b. 4 h

c. $\frac{2h}{3}$

d. $\frac{3h}{3}$

Correct Option: C

Explanation:

Let the height of the cylinder be H and its radius = r.

Then, $\pi r^2 H + \frac{1}{3}\pi r^2 h = 3 \times \frac{1}{3}\pi r^2 h$

$\Rightarrow \pi r^2 H = \frac{2}{3}\pi r^2 h$ or H = $\frac{2}{3}h$

8. If the volumes of two cones are in the ratio 1:4 and their diameters are in the ratio 4:5, then the ratio of their height is :

a. 1:5

b. 5:4

c. 5:16

d. 25:64

Correct Option: D

Explanation:

Since the diameters are in the ratio 4:5, it follows that their radii are in the ratio 4:5. Let them be 4 r and 5 r.

Let the height be h and H.

Ratio of volumes = $\frac{\frac{1}{3}\pi \times (4r)^2 \times h}{\frac{1}{3}\pi \times (5r)^2 \times H} = \frac{16h}{25H}$

$\frac{16h}{25H} = \frac{1}{4} \text{ or } \frac{h}{H} = (\frac{1}{4} \times \frac{25}{16}) = \frac{25}{64} = 25 : 64$

9. If a rightg circular cone of vertical height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is :

a. 1254 cm^2

b. 704 cm^2

c. 550 cm^2

d. 154 cm^2

Correct Option: C

Explanation :

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232$ or

$(1232 \times \frac{7}{22} \times \frac{3}{24}) = 49$

$\Rightarrow r^2 = (1232 \times \frac{7}{22} \times \frac{3}{24}) = 49$

$\Rightarrow$ r = 7 cm

Now r = 7 and h = 24, So, l = $\sqrt{7^2 + (24)^2} = \sqrt{625} = 25$ cm

Curved surface area = $\pi rl = (\frac{22}{7} \times 7 \times 25)\,cm^2 = 550 cm^2$

10. A cylindrical piece of metal of radius 2cm and height 6 cm is shaped into a cone of same radius. The height of the cone is :

a. 18cm

b. 14cm

c. 12cm

d. 8 cm

Correct Option: A

Explanation:

$\frac{1}{3}\pi \times (2)^2 \times h = \pi \times (2)^2 \times 6 \Rightarrow h = 18$ cm

11. The radii of two cylinder are in the ratio of 2:3 and their heights are in the ratio 5:3. The ratio of their volumes is :

a. 27:20

b. 20:27

c. 4 : 9

d. 9 : 4

Correct Option: B

Explanation:

Let their radii be 2 r and 3r and heights 5h and 3h respectively. Ratio of their volumes.

$$= \frac{\pi(2r)^2 \times 5h}{\pi(3r)^2 \times 3h} = \frac{20}{27} = 20 : 27$$

12. If the volume and surface area of a sphere are numerically the same, then its radius is :

a. 1 unit

b. 2 units

c. 3 units

d. 4 units

Correct Option: C

Explanation:

$\frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3$ units

13. A spherical lead ball of radius 10 cm is melted and small lead balls of radius 5 mm are made. The total number of possible small lead balls is :

a. 800

b. 125

c. 400

d. 8000

Correct Option: D

Explanatioin:

Number of balls = $\frac{\text{Volume of big ball}}{\text{Volume of small ball}} = \frac{\frac{4}{3} \times \pi \times 10 \times 10 \times 10}{\frac{4}{3} \times \pi \times 0.5 \times 0.5 \times 0.5} = 800$

14. The radii of two spheres are in the ratio 1:2. The ratio of their surface areas is :

a. 1 : 2

b. 1 : 4

c. 1 : $\sqrt{2}$

d. 3 : 8

Correct Option: B

Explanation:

Let their radii be x and 2 x.

Ratio of their surface areas = $\frac{4\pi x^2}{4\pi(2x)^2} = \frac{1}{4} = 1 : 4$

15. A right cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is :

a. 3 : 5

b. 2 : 5

c. 3 : 1

d. 1 : 3

Correct Option: D

Explanation :

Let the height of cylinder = h

and height of cone = H

Then, $\pi^2 h = \frac{1}{3}\pi r^2 H$ or $\frac{h}{H} = \frac{1}{3} = 1:3$

16. The radius of a circular cylinder is the same as that of a sphere. Their volumes are equal. The height of the cylinder is :

a. $\frac{4}{3}$ times its radius

b. $\frac{2}{3}$ times its radius

c. equal to radius

d. equal to its diameter

Correct Option: A

Explanation:

$\frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{4}{3}r$

Height = $\frac{4}{3}$ times of its radius.

17. The number of solid spheres, each of diameter 6 cm, that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm, is :

a. 3

b. 4

c. 5

d. 6

Correct Option: C

Explanation:

Let the number of spheres be x.

Then, $x \times \frac{4}{3}\pi \times (3)^3 = \pi \times (2)^2 \times 45$

or 36x = 180 or x = $\frac{180}{36} = 5$

18. The length of the wire of 0.2 mm radius that can be drawn after melting a solid copper sphere of diameter 18 cm is :

a. 24.3 m

b. 243 m

c. 2430 m

d. 24300m

Correct Option: D

Explanation:

Radius of sphere = 9 cm

Volume of sphere = $[\frac{4}{3} \times \pi(9)^3]\,cm^3 = (972\pi)cm^3$ = $\frac{1}{50}$ cm

Let the length be = x cm

Then, $972\pi = \pi \times (\frac{1}{50})^2 \times x \Rightarrow x = (972 \times 50 \times 50)$ cm

Length of wire =